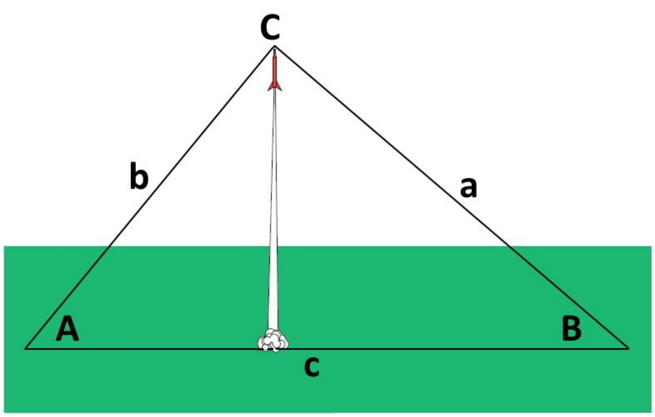
Tech Report 4

Two Station Tracking

Two Station In-line Tracking (Elevation Only)

The accuracy of your altitude tracking can be increased by adding a second tracking station. Using two elevation only trackers you can perform two station in-line tracking. In this setup the two trackers are located on each side of the launch pad, in line with the wind. It is also easier if both trackers are the same distance from the launch pad, although it is not required. Both trackers should be as far apart as possible, at least as far as the estimated altitude the rocket is expected to achieve (it is better if you can be twice the expected altitude). The farther apart the better, as this will allow for much greater accuracy on determining the altitude of the rocket.



When using two tracking stations, it is important that both tracking scopes lock in the altitude of the rocket at the same time. The use of a radio to call "mark" can help make sure both tracking scopes take their readings at the same time. Failure to lock both scopes at the same time will result in an altitude reading that is not accurate.

Method One

The first method of two station in-line tracking simply involves each tracker determining the altitude of the rocket as was done in single station tracking. The two altitudes are then averaged together to report a single altitude for the rocket.

- Station 1 Altitude = (Baseline distance) x (Tangent ∠A)
- Station 2 Altitude = (Baseline distance) x (Tangent ∠A)
- Altitude = (Station 1 Altitude + Station 2 Altitude) / 2

Example

In this scenario we have two tracking stations. The first station is 300 meters from the launch pad, while the second station is 400 meters from the pad in the opposite direction. The first tracking station records an angle of 32-degrees at apogee while the second station records an angle of 27.5 degrees.

We will start with the Station 1

- Station 1 Altitude = (Baseline distance) x (Tangent ∠A)
- Station 1 Altitude = (300 meters) x (Tangent ∠32-degrees)
- Station 1 Altitude = (300) x (0.62486935)
- Station 1 Altitude = 187.460805 meters

Next we will calculate the altitude for Station 2

- Station 2 Altitude = (Baseline distance) x (Tangent ∠A)
- Station 2 Altitude = (400 meters) x (Tangent ∠27.5-degrees)
- Station 2 Altitude = (400) x (0.52056705)
- Station 2 Altitude = 208.22682 meters

Finally we will calculate the average altitude for Station 1 and Station2

- Altitude = (Station 1 Altitude + Station 2 Altitude) / 2
- Altitude = (187.460805 + 208.22682) / 2
- Altitude = 395.687625 / 2
- Altitude = 197.8438125

Altitude = 197.84 meters

This method assumes that the rocket travels straight up, resulting in two 90-degree triangles. However, we know that this is often not what actually happens. Therefore we can use a different calculation to obtain a more accurate result.

Method Two

To achieve a more accurate calculation we use a different formula that uses sines (utilizing the symbol sin) instead of tangents. This is because we are no longer working with a right angle triangle. However, we do know the length of one side of the triangle (the baseline for both trackers) and we know two angles of the triangle. Therefore we can calculate the remainder of the triangle's measurement, and as a result the altitude of the rocket.

$$Altitude = \frac{c \times \sin(A) \times \sin(B)}{\sin(C)}$$

Example

In this scenario we will use the two tracking stations as in the first example. To calculate the baseline c we add the two baselines together (300 + 400 = 700). That still leaves the need to figure out the angle of C. This is accomplished by adding the first tracking station angle of 32-degrees to the second station angle of 27.5 degrees and then subtracting that from 180-degrees.

$$Altitude = \frac{c \times \sin(A) \times \sin(B)}{\sin(C)}$$

$$Altitude = \frac{(300+400) \times (\sin(32)) \times (\sin(27.5))}{\sin(180-(32+27.5))}$$

$$Altitude = \frac{700 \times \sin(32) \times \sin(27.5)}{\sin(120.5)}$$

$$Altitude = \frac{700 \times 0.529919 \times 0.461749}{0.861629}$$

$$Altitude = \frac{700 \times 0.244689568331}{0.861629}$$

$$Altitude = \frac{171.2826978317}{0.861629}$$

$$Altitude = 198.7893836346038$$

$$Altitude = 198.79\, meters$$

Comparing the Two Examples

As you can see the two calculations provide altitude results that are close, but not identical.

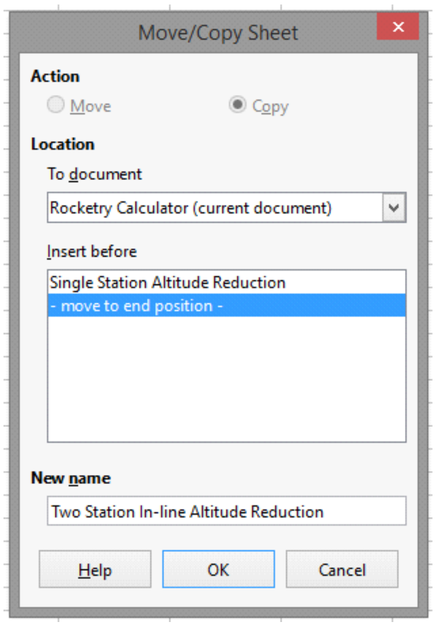
- Example 1: 197.84 meters
- Example 2: 198.79 meters

This is a difference of 0.95 meters, or nearly a 1% difference. Because of this difference, Flight Logs uses the formula described in Method 2 as it will provide a much more accurate altitude determination.

Two Station In-line Altitude Reduction

Next we are going to create a calculator that allows us to perform Two Station In-Line Altitude Reduction. We will begin by copying our Single Station Altitude Reduction sheet (from TR-3) into a new sheet. This will allow us to reuse some of the work we did earlier.

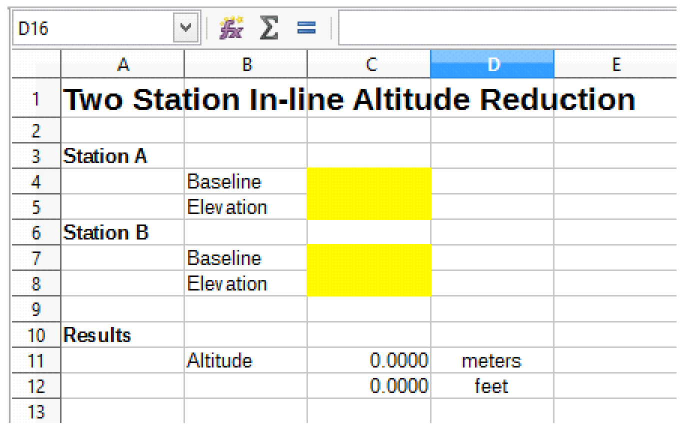
Right click on the bottom tab and select "Move or copy sheet..." from the pop-up menu. A dialog box will appear. "Copy" should already be selected. It should show the location as the "Rocketry Calculator", and "- move to end position -" should be highlighted. In the "New name" text box enter "Two Station In-line Altitude Reduction." It should look like the dialog box below. If it does, click on the "OK" button.



Move/Copy Sheet dialog box

As we begin this new calculator we can use elements from the original spreadsheet. We can copy these items and then modify them for the new calculator, saving us time and work. This is a form of code reuse, and we should use it whenever possible.

On the new spreadsheet go to row 6, right click, and select "Insert Rows Below" from the pop-up menu. Do this three times so it adds three rows to the spreadsheet. Now highlight Cells A3 through C5. This should select all of the items listed under station A. Right-click and select "Copy". Click on Cell A6, then right click and select "Paste". Change Cell A6 to Station "B". Your spreadsheet should appear



	A	B	C	D	E
1	**Two Station In-line Altitude Reduction**				
2					
3	**Station A**				
4		Baseline			
5		Elevation			
6	**Station B**				
7		Baseline			
8		Elevation			
9					
10	**Results**				
11		Altitude	0.0000	meters	
12			0.0000	feet	
13					

Two Station Altitude Reduction Setup

Like our previous spreadsheet on Single Station Tracking, we need to convert the formula into a calculation that can be performed by the spreadsheet. Here is the formula we use for two station in-line tracking:

$$Altitude = \frac{c \times \sin(A) \times \sin(B)}{\sin(C)}$$

Our next step is to calculate 'c' from our formula. This is done by adding the baseline from Station A and Station B. In our spreadsheet, the baseline for Station A is located in Cell C4, and the baseline for Station B is located in Cell C7. So our formula in Cell C11 starts like this:

= (C4 + C7)

The next part of the formula is to multiple the baseline by the Sine of ∠A. The angle for A is located in Cell C5. Like before we will use the star symbol (*) to indicate that we are going to use multiplication. We must also tell the spreadsheet to convert the Sine into Radians. So our formula now looks like this:

= (C4 + C7) * SIN(RADIANS(C5))

In the same way we will multiple the Sine of ∠B. The angle for B is located in Cell C8. Our formula now appears as follows:

= (C4 + C7) * SIN(RADIANS(C5)) *
SIN(RADIANS(C8))

This leaves the last part of our formula which has us divide the answer by the Sine of ∠C. There are several things we must do to allow the calculation to be calculated properly. First, we will need to place a parenthesis at the beginning and end of the formula as it currently stands. That way the spreadsheet will know the next operation will be on the answer to this series of calculations. To indicate that we are going to be conducting division, we use the slash symbol (/). That makes the formula appear as follows:

= ((C4 + C7) * SIN(RADIANS(C5)) *
SIN(RADIANS(C8))) /

To calculate ∠C, we must add ∠A and ∠B together, and subtract the result from 180. That part of the formula will look like this:

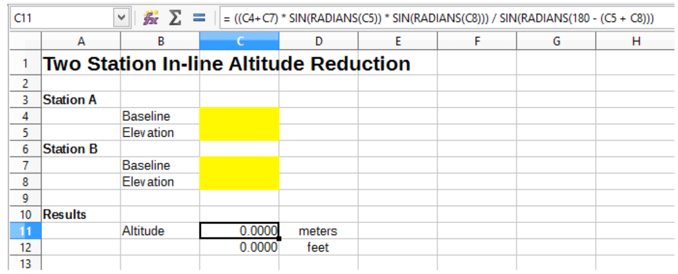
(180 – (C5 + C8))

We can now calculate the Sine of the angle just like before, with this part of the formula appearing as follows:

SIN(RADIANS(180 – (C5 + C8)))

We can add this to our full formula, which will now appear as follows:

= ((C4 + C7) * SIN(RADIANS(C5)) *
SIN(RADIANS(C8))) /
SIN(RADIANS(180 – (C5 + C8)))

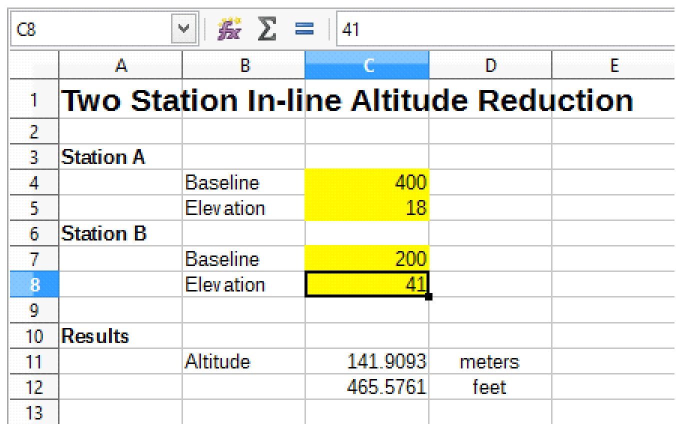


C11 = ((C4+C7) * SIN(RADIANS(C5)) * SIN(RADIANS(C8))) / SIN(RADIANS(180 - (C5 + C8)))

	A	B	C	D
1	Two Station In-line Altitude Reduction			
2				
3	Station A			
4		Baseline		
5		Elevation		
6	Station B			
7		Baseline		
8		Elevation		
9				
10	Results			
11		Altitude	0.0000	meters
12			0.0000	feet
13				

Two Station altitude reduction formula

Using the same data from the example in Chapter 10, enter it into this new calculator. If everything has been entered properly, you should see the number 141.9093 in Cell C11. Notice that even though the rows moved, the spreadsheet keep track of where it was supposed to be. The spreadsheet automatically converted meters into feet and reduced the decimal to four places.



C8 41

	A	B	C	D
1	Two Station In-line Altitude Reduction			
2				
3	Station A			
4		Baseline	400	
5		Elevation	18	
6	Station B			
7		Baseline	200	
8		Elevation	41	
9				
10	Results			
11		Altitude	141.9093	meters
12			465.5761	feet
13				

Two Station altitude reduction results

Save your work. Now you can go back and enter new entries and see how it affects the final altitude. Save your work so that you can use it over and over and never have to worry that your math is incorrect.

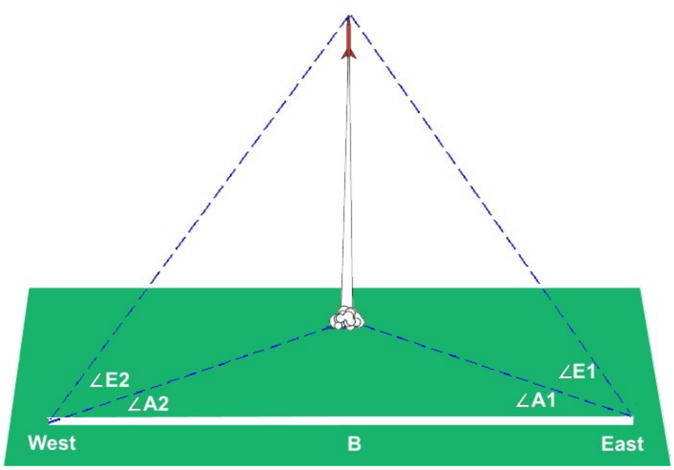
Two Station Tracking (Elevation & Azimuth)

The Two Station Elevation Azimuth method requires the use of a theodolite, an instrument that can measure both elevation and azimuth angles. The use of a theodolite allows the trackers to be more accurate in their spotting and reporting on the rocket. They do not need to be in a direct line with each other as is necessary when doing 2-station Inline tracking.

Two different calculations are used here:

- Geodesic
- Vertical Mid-point (VMP)

The formulas that are used for two-station elevation azimuth tracking in the Flight Logs software is from the National Association of Rocketry. These are the same formulas that are used for NAR competitions. They can be found on the NAR web site at https://www.nar.org/contest-flying/us-model-rocket-sporting-code/appendix/altitude-data-reduction (last accessed on August 7, 2020) and are reproduced here.



The following symbols are used in the equations:

- B is the length of the tracking baseline
- A1 and E1 are the azimuth and elevation angles reported by tracking east.
- A2 and E2 are the azimuth and elevation angles reported by tracking west.
- A is the final, reduced altitude.
- C is the closure, expressed as a fraction of the altitude. Closure ≤ 0.1 denotes a closed track. Other symbols denote common subexpressions, and are used solely for purposes of clarity.

Formula for Vertical Midpoint (VMP) Method

$$h_1 = B\frac{\sin A_2 \tan E_1}{\sin(A_1 + A_2)}$$

$$h_2 = B\frac{\sin A_1 \tan E_2}{\sin(A_1 + A_2)}$$

$$A = \frac{h_1 + h_2}{2}$$

$$C = \left|\frac{h_1 - h_2}{2A}\right|$$

Formula for Geodesic Method

$$f = \sin E_1 \sin E_2 - \cos E_1 \cos E_2 (\cos A_1 \cos A_2 - \sin A_1 \sin A_2)$$

$$d_1 = B\frac{\cos E_1 \cos A_1 + f \cos E_2 \cos A_2}{1 - f^2}$$

$$d_2 = B\frac{\cos E_2 \cos A_2 + f \cos E_1 \cos A_1}{1 - f^2}$$

$$A = \frac{d_1 d_2}{(d_1 + d_2)}(\sin E_1 + \sin E_2)$$

$$C = B\left|\frac{\cos E_2 \sin E_1 \sin A_2 - \cos E_1 \sin E_2 \sin A_1}{A\sqrt{1 - f^2}}\right|$$

If You Enjoy Rocketry, Consider Joining the NAR

If you enjoy model rocketry and projects such as the Arduino Launch Control System, Project:Icarus, The Dyna-Soar and others, then consider joining the National Association of Rocketry (NAR). The NAR is all about having fun and learning more with and about model rockets. It is the oldest and largest sport rocketry organization in the world. Since 1957, over 80,000 serious sport rocket modelers have joined the NAR to take advantage of the fun and excitement of organized rocketry.

The NAR is your gateway to rocket launches, clubs, contests, and more. Members receive the bi-monthly magazine "Sport Rocketry" and the digital NAR Member Guidebook—a 290 page how-to book on all aspects of rocketry. Members are granted access to the "Member Resources" website which includes NAR technical reports, high-power certification, and more. Finally each member of the NAR is cover by $5 million rocket flight liability insurance.

For more information, visit their web site at https://www.nar.org/