Tech Report 9

Calculating Predicted Altitude

We can use formulas to predict how high our rockets will fly. The formulas presented in this section are based on the work of Randy Culp. They are very similar to the equations developed by Len Fehskens in the 19060s, and independently by Doug Malewicki a short time after that. These equations are therefore named the "Fehskens-Malewicki equations." We have found these to be simple enough for the average person to use, while still being accurate enough to be valuable to the average rocketeer (the results are typically accurate to within 10% for most model rocket flight profiles). You can find Randy's web site "RocketMime" at http://www.rocketmime.com. It is filled with a wealth of information.

There are three primary formulas that we are going to use for altitude prediction in model rockets:
• The maximum velocity at burnout

$$v = q\frac{1 - e^{-xt}}{1 + e^{-xt}}$$

• The altitude reached at burnout

$$y_b = \frac{-m}{2k}ln\left(\frac{T - mg - kv^2}{T - mg}\right)$$

• Additional altitude attained during the coast phase, prior to ejection

$$y_c = \frac{m}{2k}ln\left(\frac{mg + kv^2}{mg}\right)$$

When all our calculations are completed, we simply add the results of altitude reached at burnout with the altitude obtained during the coasting phase to obtain our final altitude

Predicted Altitude = yb + yc

Maximum Velocity at Burnout
We begin by calculating the velocity at burnout. If you look at the equation, there are several variables listed, including the variables "q", "x", "t", and the special variable "e". We have to break down the formula into smaller sections that will provide the numbers for these variables.

To determine the variable q we use the following equation:

$$q = \sqrt{\frac{T - mg}{k}}$$

The variables "T" (thrust), "m" (mass), and "g" (gravity) we can obtain through the manufacture or by directly measuring them ourselves (and we will review them below). However, the variable "k" has its own formula that we must solve first, before we can will work through the rest of the formula for "q".

Calculating the Coefficient "k"
The coefficient k is used to determine the wind resistance on the rocket. To do this we need to look at air density, drag and the cross

section area of the rocket. The formula to calculate k is:

$$k = \frac{1}{2} \rho C_d A$$

These variables are defined as follows:
• ρ (the symbol "rho")
  This is equal to the density of air. Air density can change with changes in temperature, humidity and altitud. For model rockets, we can use a standard value of 1.225 kg/m^3. This is the value of dry air at 15^0 C (59^0 F) at sea level. This can be changed based on current weather conditions and location.
• C_d
  This is the coefficient of drag. The amount of drag on a model rocket can change based on the design of the frontal area of the model, the total amount of area, the smoothness of the finish, etc. There are some very good reports on drag and how it affects vehicles as they move through the air. For now we can use 0.75 as a good estimation for our model rockets.
• A
  This is the frontal area of the rocket. As our average rocket is a cylinder, we can use the common equation for determining the area of a circle; $A = \pi \times r^2$ or pi time the radius squared. The calculations should be in conducted in meters

As we work through the calculations we will use the Estes Goblin as an example. At the top of the column on the right is the catalog information for the model. From the information in the Estes catalog we can calculate the coefficient k.

7237 Goblin™
Length: 14.4 in. (36.6 cm)
Diameter: 1.33 in. (34 mm)
Recovery:
2 x 36 in. (91.3 cm) Streamers
Projected Altitude: 1400 ft. (427 m)
Recommended Engines: C11-3, C11-5, D12-5, D12-7

The catalog provides the following information:
• Length = 36.6 cm
  Length = 36.6 / 100 = 0.366 meters
• Diameter = 34 mm
  Diameter = 34 / 1000 = 0.034 meters

The calculations show the conversion from centimeters and millimeter measurements into meters. We can now use these numbers in the formula to find the result for k.

• First we will will calculate the frontal area of the rocket. The radius would be half the diameter, which in this case 0.017m.

$$A = \pi \times r^2$$

A = 3.141592 x 0.017^2
A = 3.141592 x 0.000289
A = 0.000907920

- We can now plug in the front area to the rest of the equation
  k = ½ ρCdA
  k = ½(1.2250) x 0.75 x 0.000907920
  k = 0.6125 x 0.75 x 0.000907920
  k = 0.000417075

Calculating the Coefficient "q" (Terminal Velocity)

Now that we have calculated "k" we can move to the next step which is to determine "q". The variable q provides you with the terminal velocity of the rocket. While a single stage model rocket doesn't have to worry about reaching terminal velocity, some of the multistage rockets will come very close.

Calculate q using the following equation:

$$q = \sqrt{\frac{T - mg}{k}}$$

To calculate the coefficient q we use the following variables:

- T = average motor thrust in Newtons
  T is the thrust of the motor in Newtons. This can be found in the documentation on the rocket motor that is used.

- m = mass of the rocket in kilograms
  The mass of the rocket will change over time, due to the burning of the propellant. As the propellant burns, the rocket will begin to get lighter. This must be taken into account with our calculations. During the calculations for velocity we will use the mass of the empty rocket (m_r) plus the mass

of the motor (m_e), then subtract half the mass of the propellant (m_p). The formula is as follows:

$$m_r + m_e - (m_p / 2)$$

We will also use the same mass figure when calculating the altitude during boost.

If you have already built the rocket, you can weigh the model to determine the actual weight. If you are building from scratch, you can add the weight of the individual components to determine the expected weight of the rocket. The weight of the motor and the propellant can be found in the documentation that comes with the motor you intend you use.

- g = acceleration of gravity = 9.81 m/s^2
  This is a constant 9.80665 m/s^2 here on the earth, and is derived from Newton's law of universal gravitation. The acceleration of gravity on other planets is different. For example, the force of gravity on the moon is 1.625 m/s^2, on Mars it is 3.711 m/s^2 while on Jupiter it is a massive 24.79 m/s^2.

- k = wind resistance and we calculated that in the previous section.

Continuing to use the Estes Goblin as an example, we can calculate the coefficient q using the information in the chart and catalog specs.

The mass of the Goblin is 70.9 grams
  m_r = 70.9g
  m_r = 70.9 / 1000
  m_r = 0.0709 kg

If we plan to fly it on a D12-3 motor the

MODEL ROCKET ENGINE PERFORMANCE CHART

Engine Type	Total Impulse	Time Delay	Est. Max. Lift Wt.		Max Thrust		Thrust Duration	Initial Weight		Propellant Weight		Diameter
	N-sec	Sec	oz	g	Newtons	lbs	Sec	oz	g	oz	g	mm
D12-3	20.00	3	14.0	396	32.90	7.4	1.60	1.57	44.5	0.85	24.2	24

motor markings and the catalog (see Motor Performance Chart at the top of this page) identifies the average thrust in Newtons (The "12" in the D12-3). We also need to determine the mass of the motor and propellant. The motor chart on the D12-3 gives us the specifications on motor mass:

m_e = 44.5g
m_e = 44.5 / 1000
m_e = 0.0445 kg

And the data on the propellant mass:

m_p = 24.2g
m_p = 24.2 / 1000
m_p = 0.0242 kg

We can now determine the average mass of the rocket during boost

$m = m_r + m_e - (m_p / 2)$
m = 0.0709 + 0.0445 − (0.0242 / 2)
m = 0.0709 + 0.0445 − 0.0121
m = 0.1033

We can now plug in the numbers into the formula to find the result for q.

$$q = \sqrt{\frac{T - mg}{k}}$$

$$q = \sqrt{\frac{12 - (0.1033 \times 9.81)}{0.000417075}}$$

$$q = \sqrt{\frac{12 - 1.013373}{0.000417075}}$$

$$q = \sqrt{\frac{10.986627}{0.000417075}}$$

$$q = \sqrt{26342.08955}$$

$$q = 162.30246$$

Calculating "x"

The variable "x", like the variables "q" and "k", we have seen thus far, has its own formula that we must work through. It is:

$$x = \frac{2kq}{m}$$

The variables in this equation have been calculated above. We can plug in our results from the earlier calculations to determine the value of x.

$$x = \frac{2 \times 0.000417075 \times 162.30246}{0.1033}$$

$$x = \frac{0.135384597}{0.1033}$$

$$x = 1.310596292$$

Calculating "t"

The variable "t" is the burn time of the motor. To determine this we divide the impulse by the thrust. This information can be found in the information on the motor package. Using our Goblin with the D12-3 motor we can calculate the burn time.

$$t = \frac{I(Impulse)}{T(Thrust)}$$

$$t = \frac{20}{12}$$

$$t = 1.6667 seconds$$

The Universal Constant "e"

In the velocity formula, we see the use of the variable "e" raised to the -xt power. In this formula the variable "e" is the universal constant "e" (sometimes called Euler's number or the exponential growth constant). It is similar to the constant π (pi) that we used earlier. The universal constant e is approximately equal to 2.718281828. In our formula for velocity at burnout we see the expression $1 - e^{-xt}$ which means that we raise the number e to the -xt power, then subtract that from 1. To solve this part of the equation we multiply -x times t, then raise e to that power.

Calculating "v"

We are now at the point where we can calculate the velocity of our Goblin rocket at burnout when using a D12-3 engine. We have calculated all of the necessary variables and can place them into the formula.

$$v = q \frac{1 - e^{-xt}}{1 + e^{-xt}}$$

$$v = 162.30246 \frac{1 - (2.712828^{-1.31060 \times 1.6667})}{1 + (2.712828^{-1.31060 \times 1.6667})}$$

$$v = 162.30246 \frac{1 - (2.712828^{-2.18438})}{1 + (2.712828^{-2.18438})}$$

$$v = 162.30246 \frac{1 - 0.11255}{1 + 0.11255}$$

$$v = 162.30246 \frac{0.88745}{1.11255}$$

$$v = 162.30246 \times 0.79767$$

$$v = 129.46380$$

The results of our calculations show that our Goblin will be traveling at 129.46 meters per second, or 289.6 miles per hour (multiply meters per second by 2.237 to convert to miles per hour). We are now ready to calculate the altitude of our Goblin at burnout as well as at the end of the coasting period.

Calculating Altitude Reached at Burnout

With the velocity at burnout now calculated, as well as the rest of the variables in the formulas for both altitude calculations, we can begin to calculate the predicted altitude for our rocket. In this calculation remember to use the rocket mass calculation that includes the propellant.

In this next formula (in the box below), there is a new function "ln" which is "Natural Logarithm". A natural logarithm "gives you the time needed to reach a certain level of growth."

Based on this calculation, the rocket will climb 125.2 meters during the boost phase. To convert meters to feet, multiple by 3.2808. which will give us an altitude of 410.76 feet.

As we move on to the next step we have already calculated the variable "v" (velocity at burnout). There are two additional variables we need to calculate next; q_a and q_b

$$t_c = \frac{\tan^{-1}\left(v/q_a\right)}{q_b}$$

Calculating "q_a"

The variable q_a uses the following formula:

$$q_a = \sqrt{\frac{mg}{k}}$$

We have seen these variables before. It is important to remember that when conducting these calculations that we use the mass of

$$y_b = \frac{-m}{2k} \ln\left(\frac{T - mg - kv^2}{T - mg}\right)$$

$$y_b = \frac{-0.1033}{2 \times 0.000417075} \ln\left(\frac{12 - (0.1033 \times 9.81) - (0.000417075 \times 129.46380^2)}{12 - (0.1033 \times 9.81)}\right)$$

$$y_b = \frac{-0.1033}{0} .00083415 \ln\left(\frac{12 - 1.013373 - 6.99054}{12 - 1.013373}\right)$$

$$y_b = \frac{-0.1033}{0.00083415} \ln\left(\frac{3.996087}{10.986627}\right)$$

$$y_b = -123.83864 \times \ln(0.36372)$$

$$y_b = -123.83864 \times -1.01137$$

$$y_b = 125.24669 meters$$

the rocket (m) where the solid rocket motor has spent all off its fuel, and all that is left is an empty motor casing. To calculate q_a:

$$q_a = \sqrt{\frac{0.0912 \times 9.81}{0.000417075}}$$

$$q_a = \sqrt{\frac{0.894672}{0.000417075}}$$

$$q_a = \sqrt{2145.11059}$$

$$q_a = 46.31534$$

Calculating "q_b"
The variable q_b uses the following formula:

$$q_b = \sqrt{\frac{gk}{m}}$$

This formula uses the same information as q_a, but performs a different calculation. To calculate q_b:

$$q_b = \sqrt{\frac{9.81 \times 0.000417075}{0.0912}}$$

$$q_b = \sqrt{\frac{0.00409150}{0.0912}}$$

$$q_b = \sqrt{0.0448629}$$

$$q_b = 0.211809$$

Calculating Coast Time
Now that we have the results for q_a and q_b, we can complete the calculation on coast time. Make sure that the result is in radians when using the arctangent ($\tan^{-1}$)

$$t_c = \frac{\tan^{-1}\left(v/q_a\right)}{q_b}$$

$$t_c = \frac{\tan^{-1}\left(129.46380/46.31534\right)}{0.211809}$$

$$t_c = \frac{\tan^{-1}(2.79527)}{0.211809}$$

$$t_c = \frac{1.22724}{0.211809}$$

$$t_c = 5.79 \text{seconds}$$

In our Goblin example, we had originally selected a D12-3 motor. Based on the coast time calculation, we should have probably selected either the D12-5 or D12-7 motor to get as close to apogee before ejection. The D12-5 would have fired the ejection charge just prior to apogee, while the D12-7 would have fired just after apogee when the rocket was starting its arc back down to earth.

Calculating Time
From Launch to Apogee
We can now calculate the flight time from launch to apogee by adding the burn time (t) to the coast time (t_a)

Flight Time = t + t_a

Flight Time = 1.6667 + 5.79

Flight Time = 7.4567 seconds

Variations in the Real World

Our calculations indicate that a Goblin using a D12 motor should attain an altitude of just under 1200 feet. Yet if we look in the Estes catalog, they show a predicted altitude of 1400 feet. Chances are if we were to build and fly a Goblin, we would not achieve the exact same results as either predicted altitude. Why the discrepancies?

There are a number of things that can account for the difference of the two projections. Estes may be using a series of calculations that are different than the ones presented here or they may take different factors into account (such as drag and air density constants).

Another thing to remember is that our calculations are performed in a sterile environment, but our rockets fly in the real world. The real world is ever changing, and the same rocket flown with the same motor 3, 4 or 5 times will have differences in every flight. Even something simple like a slightly rusty launch rod can have an effect on the overall performance of the rocket.

Within the formulas presented here there are things that can cause the predicted altitude to be different from the actual altitude seen in the field. Lets look at some of these.

Air Density

The formulas presented here used a constant number for air density (ρ=1.2250 kg/m^3, the value of dry air at 15^0 C (59^0 F) at sea level). Reality is that air density is constantly changing, depending on a number of factors. It decreases as altitude increases and it changes with variations in temperature and humidity.

If you are flying your model in Denver, Colorado, you are already at 5280 feet (one mile) above sea level (Penrose, Colorado is the home of Estes and sits approximately 5,300 feet above sea level). There you might find the air density to be 0.960 kg/m^3, a significant amount lower than that at sea level.

Temperature also plays a role. At 35^0 C (95^0 F), air density would be closer to 1.1455 kg/m^3, while a drop in temperature to -15^0 C (5^0 F) would find an air density around 1.3673 kg/m^3 at sea level.

All of these variations can have an effect on the predicted altitude of the rocket. You can see how much of an effect by simply changing out various air density values and see how the predicted altitude changes.

Motor Thrust

Much like with air density we have used a constant value for motor thrust. In reality, the amount of thrust the motor develops varies throughout the flight. We can see a graphic representation of this in a thrust curve chart.
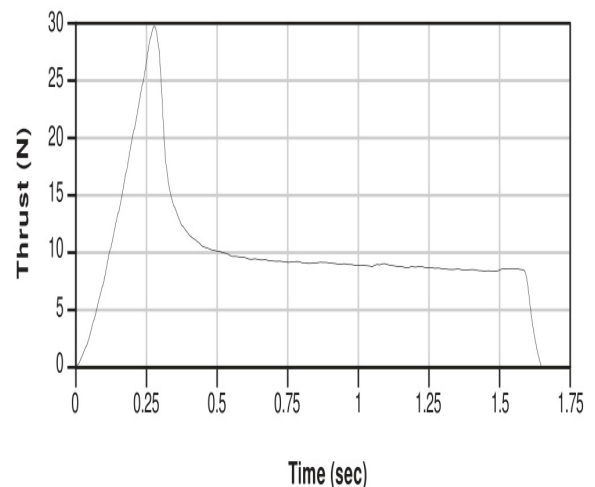
This chart shows the amount of thrust being developed by the motor through the entire burn time (the delay and ejection charge are typically not included in these charts as they do not effect the thrust being developed). A typical motor has a high peak spike thrust during the initial firing of the motor, then drops and levels off, and finally fades out as the last of the propellant is used up. The NAR thrust curve chart on the previous page shows the D12 used in our Goblin example.

When it comes to our calculations on predicted altitude it has been discovered that the use of a constant thrust figure compared to a variable thrust figure, there is little difference in the overall outcome. A much larger impact will occur by varying the total thrust of the motor you use as opposed to calculating the various changes in the motor's thrust.

Motor Mass
As before, we used a constant to define motor mass during the boost phase. We calculated the mass of the motor with only half the amount of propellant for use in our calculations. Reality again shows that as more propellant is burned, the lighter the rocket will become. The motor is at its lightest during the coasting phase when all of the propellant is gone.

With a model rocket, the mass issue is not as critical as with full scale rockets. The mass of propellant in a model rocket only accounts for 10-40% of the overall mass of the rocket. This allows us to use the constant motor weight in our calculations with a very low error rate (around 1%). Compare this to SpaceX's Falcon Heavy. The first stage has an empty mass of 25,600 kg but a propellant mass of 395,700 kg. The second stage has an empty mass of 3,900 kg and a propellant mass of 92,670 kg. Th result is a combined empty mass of 29,500kg, but a propellant mass of 488,370 kg. The propellant mass is over 16 times the empty mass of the rocket.

Wind Resistance
We also use a number of constants for wind resistance. The calculations assume a basic model construction of a single main body tube and some fins (which is why only the frontal area of the rocket is calculated). A model that has multiple body tubes, large fins that are nearly like wings, or other objects that are placed into the air stream can significantly increase the amount of drag on the rocket.

We used a Coefficient of Drag as a constant 0.75. While drag can be increased as noted above, it can also be reduced. The use of a smooth, glass-like finish, a more aerodynamic nosecone, the use of elliptical fins, even the use of a boat-tail can all reduce drag. Such a a well constructed and finished model could have a C_d as low as 0.3. However, a model rocket that is poorly constructed, a rough surface finish, etc may have a C_d of 2.0 or more.

Like the other constants we used throughout the formulas, the use of a constant Coefficient of Drag of 0.75 is a good compromise between accuracy and ease of use. Adjustments to the C_d may result in a more accurate projection.

The formulas presented in this section are designed to provide a good baseline from which to work. As you become more interested in the various aspects of model rocket aerodynamics, additional research will show how you can obtain more accurate results. You can then begin to adjust the formulas to match your findings.

Even without additional research, you can build and fly a model rocket and compare the results in the field with the results provided by these formulas. You can then try to adjust certain variables to see if you can obtain more accurate results from the formula. You can also see if these changes are consistent across a variety of rockets flown in the field.

Predicted Altitude for Multistage Rockets

When we look at the predicted altitude calculations for a multistage rocket, we find that the majority of the formulas are going to be exactly the same as they were for the single stage rocket. There are, however, a couple of important differences.

• The calculation for the boost stage of flight is the same for first stage of a multistage rocket as it is for a single stage rocket.

• You have to calculate boost equations for each stage.

• The second and subsequent stages must incorporate the velocity at the end of the previous stage. We must include the end velocity of the first stage as the initial velocity for the second stage, and the end velocity of the second stage as the initial velocity of the third stage.

• We must calculate the terminal velocity for the rocket under thrust. With a multistage rocket it is possible that you may come close or reach terminal velocity during the boost stages.

You also need to add the altitude for each stage during its boost phase and the altitude for the coast stage in the last stage only. Remember the booster stage(s) do not coast but ignite as soon as the previous stage finishes.

Calculate the Booster Stage

The booster stage or first stage is calculated just like you would a single stage rocket. The only differences are you will not calculate coast time (as that occurs following the last stage of the flight) and your mass calculation must include the mass of all the booster stages. For a three stage rocket, the initial mass calculation would appear as follows:

$$m = m_{rocket} + m_{third\ motor} + m_{second\ motor} + m_{first\ motor} - (m_{first\ propellant}/2)$$

Calculate for Each Stage

After you have calculated the first stage, some of the calculations will change slightly to account for the multiple stages. The formulas for "q", "x" and "t" do not change, but they must be calculated for each stage based on new values.

Changing Rocket Mass

Once the first stage booster ceased firing, it ignites the second stage booster and the first stage drops away. We no longer need to calculate the first stage and can remove it from our equations.

Our formula for the mass of the rocket while the second stage is firing would appear as follows:

$$m = m_{rocket} + m_{third\ motor} + m_{second\ motor} - (m_{second\ propellant}/2)$$

When you perform the calculations for each stage, you need to make sure you remove the weight from the previous stage that is no longer attached to the rocket.

Changing Rocket Velocity

The other change that you must take into account is the changing velocity of the rocket as each stage ignites, burns and drops off. To calculate these effects we use a new formula for each upper stage to calculate the value of "s". That formula is as follows:

$$s = \frac{q + v_o}{q - v_o}$$

The variable v_O is the velocity of the previous stage. We can now use this in our calculations for new velocity of the rocket as well as the altitude attained during the current stage burn.

The new formula for velocity is:

$$v = q\frac{s - e^{-xt}}{s + e^{-xt}}$$

This new calculation provides the velocity at the end of the burn.

Calculate Additional Rocket Altitude

The formula to calculate the altitude for each stage is basically the same as before, except it now takes into account the changes in velocity as each stage completes it's burn.

You will also notice that the variable y_b has become y_n as there are multiple stages. So for a three stage rocket you will have to perform calculations for y_1, y_2 and y_3. As before, v_O refers to the velocity of the previous stage.

The updated formula is as follows:

$$y_n = \frac{-m}{2k}\ln\left(\frac{T - mg - kv^2}{T - mg - kv_o^2}\right)$$

The altitude that you calculate with this formula is only for that stage of the burn. Therefore to determine the complete altitude you must calculate the altitude achieved for each stage burn as well as the coasting altitude. Once you have these individual altitudes, you add them together to determine the total altitude achieved.

Those with an eye for math will notice that if $v_O = 0$, the equations are the same as for the single stage rocket.

The coasting calculations are the same as for the single stage rocket. It does not change.

Parachute Calculations

In this section we are going to perform two calculations on parachute performance:

• Determining how large a parachute you will need for your rocket.

• Determine the descent velocity of parachute.

Calculating Parachute Size

The first formula we will discuss is how to determine the proper size of the parachute needed for a rocket. The formula that we will use is:

$$D = \sqrt{\frac{8mg}{\pi \rho C_d v^2}}$$

Most of the variables used here are the same as in our altitude prediction calculations.

- C_d = is the coefficient of drag. For our parachute we will use either 0.75 for parachutes made out of a flat piece of plastic sheet (as is common on most model rocket kits). You could also use 1.5 for a true round canopy parachute.

- D = diameter of the parachute in meters

- g = acceleration of gravity = 9.81 m/s^2

- m = rocket mass in kilograms. Since we are calculating parachute size, the parachute has been deployed, any boosters have separated, and the main rockets motor has expelled all its propellant. Therefore you want to use the rocket mass that includes the empty rocket and empty motor casing, with no propellant.

- π = 3.1415926

- ρ = the density of air. We will continue to use a value of 1.2250 kg/m^3, the value of dry air at 15^0 C (59^0 F) at sea level.

- v = velocity. Here we want our rocket to land softly so we will start with a speed of 3 meters per second.

We will continue to use our Goblin rocket and calculate what size parachute it will need.

$$D = \sqrt{\frac{8 \times 0.0912 \times 9.81}{3.1415926 \times 1.2250 \times 0.75 \times 3^2}}$$

$$D = \sqrt{\frac{7.157376}{25.97704}}$$

$$D = \sqrt{0.275527}$$

$$D = 0.52491 meters$$

To convert our answer in meters to inches (which is the way parachute sizes are listed in most rocket kits), will will multiple 0.52491 meters by 39.370. This gives a result of 20.6 inches. Since the Goblin comes with two streamers instead of a parachute, we could replace the streamers with a 21-inch parachute.

Calculating Parachute Descent Rate

Sometimes we have a rocket and a parachute, but we are not sure if it is large enough to slow the rocket down sufficiently for a safe landing. We could also be worried that the parachute is too big, causing the rocket to drift away from the flying field. Using a variation of the formula for parachute size above, we can calculate the descent rate of a rocket and it's parachute. The formula we will use in this case is:

$$v = \sqrt{\frac{8mg}{\pi \rho C_d D^2}}$$

All of the variables are the same as the formula above, except in this case we are calculating the descent velocity because we know the parachute diameter.

Let's go back to our Goblin rocket with its two streamers. It is a calm day and we are on a large field, so we decide we want to use a parachute instead. We have two different size parachutes in our range box; a 12-inch parachute and a 24-inch parachute. We can calculate the descent rate for both parachutes and see which one will best suit our needs.

Calculations for the 12-inch Parachute

First, convert inches into meters

$$12 / 39.370 = 0.30480$$

Now we can calculate the descent rate for our 12-inch parachute.

$$v = \sqrt{\frac{8 \times 0.0912 \times 9.81}{3.1415926 \times 1.2250 \times 0.75 \times 0.30480^2}}$$

$$v = \sqrt{\frac{7.157376}{0.268149}}$$

$$v = \sqrt{26.69179}$$

$$v = 5.166 \text{ meters per second}$$

This is nearly twice as fast as our target goal of 3 meters per second. Now let's see how well the 24-inch parachute performs.

Calculations for the 24-inch Parachute

Once again, convert inches into meters

· $24 / 39.370 = 0.6096$

Next we calculate the descent rate.

$$v = \sqrt{\frac{8 \times 0.0912 \times 9.81}{3.1415926 \times 1.2250 \times 0.75 \times 0.60960^2}}$$

$$v = \sqrt{\frac{7.157376}{1.0726}}$$

$$v = \sqrt{6.67292}$$

$$v = 2.583 \text{ meters per second}$$

The 24-inch parachute at 2.6 m/s is the closest to our goal of 3 m/s descent rate. The 12-inch parachute, with a descent rate of 5.1 m/s, doesn't slow the rocket enough to prevent damage upon landing. We would have to keep an eye on our rocket when using the 24-inch parachute, as it would be more susceptible to drifting in a breeze, but the chance of inflicting damage upon landing is significantly reduced.

Total Flight Time

With all of the calculations we have performed thus far we can now calculate the total flight time. The formula we will use is:

$$t_{tft} = t_b + t_c + t_d$$

The variables we are using are
t_{tft} = Total flight time
t_b = Boost time
t_c = Coast time
t_d = Descent time

Earlier we calculated the boost time and the coast time. To calculate the descent time, we will use our calculated altitude and divide that by our calculated descent rate

t_d = predicted altitude / descent rate

Calculate Descent Time

Using our Goblin example rocket, we had previously calculated an altitude of 363.1 meters. If we use a 24-inch parachute, the descent rate is 2.6 m/s. We can place these numbers into the formula to calculate descent time:

t_d = predicted altitude / descent rate
t_d = 363.1 / 2.6
t_d = 139.65 seconds

Calculate Total Flight Time

We now have all of the numbers we need to calculate total flight time:

$t_{tft} = t_b + t_c + t_d$
t_{tft} = 1.6667 + 5.79 + 139.65
t_{tft} = 147.1067 seconds.

If we divide this by 60, we can determine the flight time in minutes:

147 / 60 = 2.45 minutes or
= 2 minutes 27 seconds.

If You Enjoy Rocketry, Consider Joining the NAR

If you enjoy model rocketry and projects such as the Arduino Launch Control System, then consider joining the National Association of Rocketry (NAR). The NAR is all about having fun and learning more with and about model rockets. It is the oldest and largest sport rocketry organization in the world. Since 1957, over 80,000 serious sport rocket modelers have joined the NAR to take advantage of the fun and excitement of organized rocketry.

The NAR is your gateway to rocket launches, clubs, contests, and more. Members receive the bi-monthly magazine "Sport Rocketry" and the digital NAR Member Guidebook—a 290 page how-to book on all aspects of rocketry. Members are granted access to the "Member Resources" website which includes NAR technical reports, high-power certification, and more. Finally each member of the NAR is cover by $5 million rocket flight liability insurance.

For more information, visit their web site at https://www.nar.org/