Tech Report 9s

Predicted Altitude Caculations for Farside-X

In Tech Report 9, "Calculating Predicted Altitude", we presented a series of calculations that can be used to develop a complete flight profile prediction for a multistage rocket, from launch to touchdown. In this Supplement to Tech Report 9 (the reason for the "s" in the report number) we are going to develop the flight profile for a multi-stage rocket using these calculations. In this exercise we will use the Fireside-X. Semroc did make a reproduction of the Estes Astron kit and it can still be found on some hobby sites. You can download the RockSim file from Rocket Reviews (https://www.rocketreviews.com/rocksim-library.html) where there are two versions listed. These RockSim files can be viewed in OpenRocket as well. It is also a model that can be cloned without much difficulty.

To perform our calculations we will be using the model kit data from the 1970 Estes catalog. This will provide us with the information we need.

As we go through each stage of the calculations, we will show you all of our work. You won't just see the basic formula and the result. We will show you each step so you can see how we obtained the result, and if your results are different from ours, you can easily see where the difference occurs (we checked our math several times but it is possible we made a mistake. If you believe you have found an error in these calculations, please let us know).

You will also notice that after this initial page the rest of the report is in a single column format that expands across the entire page. This is due to the fact that some of the equations used in this report are rather long. We felt it was more important for you to see the full equations in a normal size font than to try and squeeze them down to fit into a two-column format.

We have downloaded the RockSim files and can view them in OpenRocket, we can also perform flight simulations. We can then compare the results of our calculations to the findings in OpenRocket. If desired, you can take it to the next level and actually build, fly and collect the data on how well the rocket performs. Those results can be compared to both our calculations and the OpenRocket results. Before we can start to compare anything, however, we start conducting our own analysis and performing the calculations.

The FarSide X Rocket

Here is the information for the Fireside X model rocket from the 1970 Estes catalog (seen on the right):

- Launch Vehicle Data
  - Length = 63.5 cm (0.635 m)
  - Diameter = 41.6 mm (0.0416 m)
  - Mass (Empty) = 71 g (0.071 kg)

The values for motor performance will use the Estes documentation in the 2016 catalog, which is the same data we used for the Gremlin calculations in TR-9. This data can be seen in the chart at the bottom of the page.

- First Stage = B6-0
  - Average Motor Thrust = 6 Newtons
  - Total Impulse = 5 Newton-seconds
  - Motor Mass = 15.7 g (0.0157 kg)
  - Propellant Mass = 6.5g (0.0065 kg)

- Second Stage = C6-0
  - Average Motor Thrust = 6 Newtons
  - Total Impulse = 10 Newton-seconds
  - Motor Mass = 21.4 g (0.0214 kg)
  - Propellant Mass = 12.2 g (0.0122 kg)

Third Stage = C6-7
  - Average Motor Thrust = 6 Newtons
  - Total Impulse = 10 Newton-seconds
  - Motor Mass = 24.3 g (0.0243 kg)
  - Propellant Mass = 12.2 g (0.0122 kg)

Astron Farside X 1970 Estes Catalog

Prod. No.	Engine Type	Total Impulse	Time Delay	Max. Lift Wt.		Max. Thrust		Thrust Duration	Initial Weight		Propellant Weight	
		N-sec	Sec	oz	g	Newtons	lbs	Sec	oz	g	oz	g
BOOSTER STAGE ENGINES												
1608	B6-0	5.00	None	4.0	113	12.1	2.7	0.8	0.55	15.7	0.23	6.5
1616	C6-0	10.00	None	4.0	113	15.3	3.4	1.6	0.76	21.4	0.43	12.2
UPPER STAGE ENGINES												
1615	C6-7	10.00	7	2.5	71	15.3	3.4	1.6	0.85	24.3	0.43	12.2

Solid Rocket Motor Specifications, 2016 Estes Catalog

Formula Constants

As with the Gremlin calculations, we will use the same constants as we did in the previous equations and will work through each equation for each stage.

- Air density (ρ = 1.2250 kg/m^3),
- Coefficient of Drag (C_d = 0.75), and
- Acceleration of gravity (g = 9.81 m/s^2)
- Pi (π = 3.1415926)
- e = 2.7182818
- Decimals are rounded to seven places

You will also notice that y_b is no longer used. We now have two booster stages as well as the sustainer. Instead of y_b you will find each stage listed as y_1, y_2 and y_3. The coasting calculations will stay as y_c.

First Stage Calculations

We begin with the first stage booster of the rocket and will include all of the calculations for this stage and each subsequent stage of the rocket.

Calculate First Stage Wind Resistance

Calculate the frontal area “A”.

$A = \pi \times r^2$
$A = 3.1415926 \times (0.0416 / 2)^2$
$A = 3.1415926 \times 0.0.02082$
$A = 3.1415926 \times 0.00043264$
$A = 0.0013591$

Now that we have the frontal are we can calculate the wind resistance “k”.

$k = ½ \rho C_d A$
$k = ½ \times 1.2250 \times 0.75 \times 0.0013591$
$k = 0.6125 \times 0.75 \times 0.0013591786$
$k= 0.0006243$

Calculate First Stage “q” Coefficient

To calculate “q” we use the following equation.

$$q = \sqrt{\frac{T - mg}{k}}$$

The calculation for “m” in this case is the mass of the empty rocket, plus the mass of the motor in each stage except the first stage, which will be the mass of the motor minus half the propellant.

$$m = m_{rocket} + m_{third} + m_{second} + (m_{first} - (m_{firstpropellant}/2))$$

$$m = 0.071 + 0.0243 + 0.0214 + (0.0157 - (0.0065 / 2))$$

$$m = 0.071 + 0.0243 + 0.0214 + (0.0157 - 0.00325)$$

$$m = 0.12915 \text{ kg}$$

Having determined “m” for Stage 1 we can now calculate “q”.

$$q = \sqrt{\frac{6 - (0.12915 \times 9.81)}{0.0006243}}$$

$$q = \sqrt{\frac{6 - 1.2669615}{0.0006243}}$$

$$q = \sqrt{\frac{4.7330385}{0.0006243}}$$

$$q = \sqrt{7581.352715}$$

$$q = 87.0709637$$

Calculate First Stage “x” Coefficient

Now calculate “x” for Stage 1 using the following equation.

$$x = \frac{2\,kq}{m}$$

$$x = \frac{2 \times 0.0006243 \times 87.0709637}{0.12915}$$

$$x = \frac{0.1087168}{0.12915}$$

$$x = 0.8417871$$

Calculate First Stage Burn Time "t"

Next calculate the Stage 1 burn time "t"

$$t = \frac{I}{T}$$

$$t = \frac{5}{6}$$

$$t = 0.8333333$$

Calculating First Stage Velocity "v1"

Now calculate the velocity at burnout for Stage 1.

$$v_1 = q\,\frac{1 - e^{-xt}}{1 + e^{-xt}}$$

$$v_1 = 87.0709637\,\frac{1 - 2.7182818^{(-0.8417871 \times 0.8333333)}}{1 + 2.7182818^{(-0.8417871 \times 0.8333333)}}$$

$$v_1 = 87.0709637\,\frac{1 - 2.7182818^{-0.7014892}}{1 + 2.7182818^{-0.7014892}}$$

$$v_1 = 87.0709637\,\frac{1 - 0.4958463}{1 + 0.4958463}$$

$$v_1 = 87.0709637\,\frac{0.5041537}{1.4958463}$$

$$v_1 = 87.0709637 \times 0.3370358$$

$$v_1 = 29.3460319$$

Calculating First Stage Altitude Reached at Burnout “y1”

We are now at the end of the Stage 1 calculations and can calculate the altitude of Stage 1 at burnout. As some of these calculations are rather lengthy, they will cover the entire page.

$$y_1 = \frac{-m}{2k} \ln\left(\frac{T - mg - kv_1^2}{T - mg}\right)$$

$$y_1 = \frac{-0.12915}{2 \times 0.0006243} \ln\left(\frac{6 - (0.12915 \times 9.81) - (0.0006243 \times 29.3460319^2)}{6 - (0.12915 \times 9.81)}\right)$$

$$y_1 = \frac{-0.12915}{0.0012486} \ln\left(\frac{6 - 1.2669615 - 0.5376407}{6 - 1.2669615}\right)$$

$$y_1 = \frac{-0.12915}{0.0012486} \ln\left(\frac{4.1953978}{4.7330385}\right)$$

$$y_1 = -103.4358481 \times \ln(0.8864069)$$

$$y_1 = (-103.4358481) \times (-0.1205792)$$

$$y_1 = 12.4722118 \; meters$$

Second Stage Calculations

Now that we have completed the calculations for the first stage, we need to perform nearly the same calculations again. We do not need to recalculate “A” or “k” as they will remain constant in all of the remaining computations. Even when the remaining formulas don’t change, the data elements will change (such as a different motor resulting in a different impulse and mass).

Additionally, with the remaining stages we need to include the ending velocity of the previous stage as the beginning velocity of the next stage. We identify the velocity of the previous stage at burn out with the variable “v_O”. Additionally as each stage completes its burn and drops off, the rocket mass will change and get lighter so we must account for this change as well.

Calculate Second Stage "q" Coefficient

With a multistage rocket, the weight becomes less as each booster stage completes it's burn and separates from the rocket. The empty weight of each booster stage is about 3 grams. When compared to the weight of the rocket motor used in the booster stage, this weight becomes negligible. As before, we cannot calculate the value of q until we update the mass of the rocket in the new configuration. The calculation for "m" in this case is the mass of the empty rocket, plus the mass of the motor for the sustainer, and the mass of the motor minus half the propellant for the second stage.

$$m = m_{rocket} + m_{third} + (m_{second} - (m_{second}propellant/2))$$

$$m = 0.071 + 0.0243 + 0.0214 - (0.0122 / 2)$$

$$m = 0.1106$$

With the new mass determined for the rocket we can now complete the calculation for q

$$q = \sqrt{\frac{T - mg}{k}}$$

$$q = \sqrt{\frac{6 - (0.1106 \times 9.81)}{0.0006243}}$$

$$q = \sqrt{\frac{6 - 1.084986}{0.0006243}}$$

$$q = \sqrt{\frac{4.915014}{0.0006243}}$$

$$q = \sqrt{7872.839981}$$

$$q = 88.7290256$$

Calculate Second Stage "x" Coefficient

While the formula is the same as with Stage 1, there is a new mass (m) and terminal velocity (q), so the outcome will be different.

$$x = \frac{2\,kq}{m}$$

$$x = \frac{2 \times 0.0006243 \times 88.7290256}{0.1106}$$

$$x = \frac{0.1107871}{0.1106}$$

$$x = 1.0016917$$

Calculate Second Stage Burn Time "t"

$$t = \frac{I}{T}$$

$$t = \frac{10}{6}$$

$$t = 1.6666667 \; seconds$$

Calculate Second Stage "s" Coefficient

With the multistage rocket we must take into consideration the velocity of the rocket of the previous stage at burnout. The variable "v_1" seen in the formula below is the velocity of the previous stage. Here is the new formula to include this data.

$$s = \frac{q + v_1}{q - v_1}$$

$$s = \frac{88.7290256 + 29.3460319}{88.7290256 - 29.3460319}$$

$$s = \frac{118.0750575}{59.3829937}$$

$$s = 1.9883649$$

Calculating Second Stage Velocity "v_2"

To account for the first stage velocity, the formula has changed slightly. However, if $v_1 = 0$ then the formula correctly reverts back to the original as seen in the first stage calculations.

$$v_2 = q\,\frac{s - e^{-xt}}{s + e^{-xt}}$$

$$v_2 = 88.7290256\,\frac{1.9883649 - \left(2.7182818^{(-1.0016917 \times 1.6666667)}\right)}{1.9883649 + \left(2.7182818^{(-1.0016917 \times 1.6666667)}\right)}$$

$$v_2 = 88.7290256\,\frac{1.9883649 - 2.7182818^{-1.6694862}}{1.9883649 + 2.7182818^{-1.6694862}}$$

$$v_2 = 88.7290256\,\frac{1.9883649 - 0.1883438}{1.9883649 + 0.1883438}$$

$$v_2 = 88.7290256\,\frac{1.8000211}{2.1767087}$$

$$v_2 = 88.7290256 \times 0.8269463$$

$$v_2 = 73.3741394\ \textit{meters per second}$$

Calculating Second Stage Altitude Reached at Burnout "y_2"

The calculation for the second stage altitude is nearly the same as the first stage, except that it includes allowances for the velocity of the first stage (v_1). Again, if v_1 is found to be 0, then the formula rightly reverts back to the original formula, so both formulas are mathematically sound.

$$y_2 = \frac{-m}{2k}\ln\left(\frac{T - mg - kv_2^2}{T - mg - kv_1^2}\right)$$

$$y_2 = \frac{-0.1106}{2 \times 0.0006243}\ln\left(\frac{6 - (0.1106 \times 9.81) - (0.0006243 \times 73.3741394^2)}{6 - (0.1106 \times 9.81) - (0.0006243 \times 29.3460319^2)}\right)$$

$$y_2 = \frac{-0.1106}{0.0012486}\ln\left(\frac{(6 - 1.084986) - (0.0006243 \times 5383.764333)}{(6 - 1.084986) - (0.0006243 \times 861.1895883)}\right)$$

$$y_2 = \frac{-0.1106}{0.0012486}\ln\left(\frac{4.915014 - 3.3610841}{4.915014 - 0.5376407}\right)$$

calculation continued next page

$$y_2 = \frac{-0.1106}{0.0012486} \ln\left(\frac{1.5539299}{4.3773733}\right)$$

$$y_2 = -88.5792087 \times \ln(0.3549914)$$

$$y_2 = -88.5792087 \times -1.0356617$$

$$y_2 = 91.7380939\ meters$$

Third Stage Calculations

We now have the altitude for the first and second stage, but we still need to perform the same calculations once again for the third or sustainer stage. These calculations will again be very much like the second stage calculations.

Calculate Third Stage "q" Coefficient

As with the second stage we cannot calculate the value of "q" until we update the mass of the rocket in the new configuration. The calculation for "m" in this case is the mass of the empty rocket, plus the mass of the motor for the third stage sustainer, minus half the propellant.

$m = m_{rocket} + m_{third} – (m_{third}propellant/2)$

m = 0.071 + 0.0243 – (0.0122 / 2)
m = 0.0892

With the new mass determined for the rocket we can now complete the calculation for "q".

$$q = \sqrt{\frac{T - mg}{k}}$$

$$q = \sqrt{\frac{6 - (0.0892 \times 9.81)}{0.0006243}}$$

$$q = \sqrt{\frac{6 - 0.875052}{0.0006243}}$$

$$q = \sqrt{\frac{5.124948}{0.0006243}}$$

$$q = \sqrt{8209.111004}$$

$$q = 90.6041445$$

Calculate Third Stage "x" Coefficient

To calculate "x" use the same equation as before but with a new mass (m) and terminal velocity (q).

$$x = \frac{2\,kq}{m}$$

$$x = \frac{2 \times 0.0006243 \times 90.6041445}{0.0892}$$

$$x = \frac{0.1131283}{0.0892}$$

$$x = 1.2682545$$

Calculate Third Stage "s" Coefficient

Both of the booster stages have burned their fuel and fallen from the rocket. The variable "v_2" seen in the formula below is the velocity of the second stage. Here is the formula including this data:

$$s = \frac{q + v_2}{q - v_2}$$

$$s = \frac{90.6041445 + 73.3741394}{90.6041445 - 73.3741394}$$

$$s = \frac{163.9782839}{17.2300051}$$

$$s = 9.5170189$$

Calculating Third Stage Velocity "v_3"

We need to account for the second stage velocity like we did previously with the first stage velocity. The formula is exactly the same as for the second stage, only the numbers are different.

$$v_3 = q\,\frac{s - e^{-xt}}{s + e^{-xt}}$$

$$v_3 = 90.6041445\,\frac{9.5170189 - \left(2.7182818^{(-1.2682545 \times 1.6666667)}\right)}{9.5170189 + \left(2.7182818^{(-1.2682545 \times 1.6666667)}\right)}$$

calculation continued next page

$$v_3 = 90.6041445\,\frac{9.5170189-\left(2.7182818^{-2.1137575}\right)}{9.5170189+\left(2.7182818^{-2.1137575}\right)}$$

$$v_3 = 90.6041445\,\frac{9.5170189-0.1207833}{9.5170189+0.1207833}$$

$$v_3 = 90.6041445\,\frac{9.3962356}{9.6378022}$$

$$v_3 = 90.6041445 \times 0.9749355$$

$$v_3 = 88.3331969 \textit{ meters per second}$$

Remember that the formula for "q" provides the terminal velocity of the rocket. For the third stage, the terminal velocity was 90.6 meters per second. Notice that the third stage sustainer reaches a velocity of 88.3 meters per second, only 2.3 meters per second from terminal velocity.

Calculating Third Stage Sustainer Altitude Reached at Burnout "y_3"

The calculation for the third stage sustainer stage altitude is the same as the second stage, except that includes allowances for the velocity of the second stage (v_2).

$$y_3 = \frac{-m}{2k}\ln\left(\frac{T-mg-kv_3^2}{T-mg-kv_2^2}\right)$$

$$y_3 = \frac{-0.0892}{2\times 0.0006243}\ln\left(\frac{6-(0.0892\times 9.81)-(0.0006243\times 88.3331969^2)}{6-(0.0892\times 9.81)-(0.0006243\times 73.3741394^2)}\right)$$

$$y_3 = \frac{-0.0892}{0.0012486}\ln\left(\frac{(6-0.875052)-(0.0006243\times 7802.753675)}{(6-0.875052)-(0.0006243\times 5383.764333)}\right)$$

$$y_3 = \frac{-0.0892}{0.0012486}\ln\left(\frac{5.124948-4.8712591}{5.124948-3.3610841}\right)$$

$$y_3 = \frac{-0.0892}{0.0012486}\ln\left(\frac{0.2536889}{1.7638639}\right)$$

$$y_3 = -71.4400128 \times \ln(0.1438257)$$

$$y_3 = (-71.4400128)\times(-1.9391531)$$

$$y_3 = 138.5331223 \textit{ meters}$$

Calculating Additional Altitude of Third Stage During Coasting

The calculation for the altitude attained during the coast phase of the three stage rocket flight is identical to the single stage rocket flight. However, once again we need to recalculate the mass of the rocket, as now there is no propellant left. The formula is similar:

$m = m_{rocket} + m_{third} – m_{third}propellant$

$m = 0.071 + 0.0243 – 0.0122$

$m = 0.0831$

This is the mass that will be used during the coasting phase. Now we can calculate the altitude gained during the coast phase of flight

$$y_c = \frac{m}{2\,k} \ln\left(\frac{mg + kv_3^2}{mg}\right)$$

$$y_c = \frac{0.0831}{2 \times 0.0006243} \ln\left(\frac{(0.0831 \times 9.81) + (0.0006243 \times 88.3331969^2)}{(0.0831 \times 9.81)}\right)$$

$$y_c = \frac{0.0831}{0.0012486} \ln\left(\frac{(0.815211) + (0.0006243 \times 7802.753675)}{(0.815211)}\right)$$

$$y_c = \frac{0.0831}{0.0012486} \ln\left(\frac{0.815211 + 4.8712591}{0.815211}\right)$$

$$y_c = \frac{0.0831}{0.0012486} \ln\left(\frac{5.6864701}{0.815211}\right)$$

$$y_c = 66.5545411 \times \ln(6.9754580)$$

$$y_c = 66.5545411 \times 1.942398$$

$$y_c = 129.2754075 \; meters$$

Calculating Total Altitude

We can now determine the total predicted altitude for our Farside X rocket. Simply add together the two booster altitudes (y_1 and y_2), the third stage sustainer altitude (y_3), and the third stage coasting altitude (y_c).

$$\text{Total Altitude} = y_1 + y_2 + y_3 + y_c$$

$$\text{Total Altitude} = 12.4722118 + 91.7380939 + 138.5331223 + 129.2754075$$

$$\text{Total Altitude} = 372.0188355 \text{ meters}$$

Third Stage Sustainer Coast Time

To calculate the coast time for the multistage rocket we use the same formula as for a single stage rocket.

$$t_c = \frac{\tan^{-1}(v_3/q_a)}{q_b}$$

Calculating "q_a"

As before we start by calculating the mass of the rocket. This includes the rocket and the empty motor casing. We just did this calculation when we calculated the mass for the third stage altitude formula. So we know our empty third stage mass is 0.0831. To calculate q_a:

$$q_a = \sqrt{\frac{mg}{k}}$$

$$q_a = \sqrt{\frac{0.0831 \times 9.81}{0.0006243}}$$

$$q_a = \sqrt{\frac{0.815211}{0.0006243}}$$

$$q_a = \sqrt{1305.800096}$$

$$q_a = 36.1358561$$

Calculating "q_b"

The variable q_b also uses the same formula as the single stage rocket:

$$q_b = \sqrt{\frac{gk}{m}}$$

$$q_b = \sqrt{\frac{9.81 \times 0.0006243}{0.0831}}$$

$$q_b = \sqrt{\frac{0.0061244}{0.0831}}$$

$$q_b = \sqrt{1305.800096}$$

$$q_b = 36.1358561$$

Calculating Coast Time

Now that we have the results for q_a and q_b, we can complete the calculation on coast time. Make sure that the result is in radians when using the arctangent ($\tan^{-1}$)

$$t_c = \frac{\tan^{-1}(v_3/q_a)}{q_b}$$

$$t_c = \frac{\tan^{-1}(88.3331969/36.1358561)}{0.2714760}$$

$$t_c = \frac{\tan^{-1}(2.4444750)}{0.2714760}$$

$$t_c = \frac{1.182482}{0.2714760}$$

$$t_c = 4.3557515\ seconds$$

Calculating Time From Launch to Apogee

In the same way we calculated total altitude we can now calculate the flight time from launch to apogee by adding together each motor's burn time (t_1, t_2 and t_3) along with the coast time (t_c)

Flight Time = $t_1 + t_2 + t_3 + t_c$

Flight Time = 0.8333333 + 1.6666667 + 1.6666667 + 4.3557515

Flight Time = 8.5224182 seconds

Calculating Parachute Size

Calculating the parachute size for a multistage rocket is exactly the same as for a single stage rocket. It is important to remember to only include the mass of the third stage sustainer and the empty rocket motor, as we did when calculating the third stage altitude.

As with the single stage rocket we have selected a descent rate (v_d) of 3 meters per second. For our parachute we will use a C_d of 0.75. This is typical when using a flat plastic parachute that is common on most model rocket kits.

$$D=\sqrt{\frac{8mg}{\pi \rho C_d v_d^2}}$$

$$D=\sqrt{\frac{8\times 0.0831\times 9.81}{3.1415926\times 1.2250\times 0.75\times 3^2}}$$

$$D=\sqrt{\frac{6.521688}{25.9770438}}$$

$$D=\sqrt{0.2510558}$$

$$D=0.5010547\ meters$$

$$D=19.7265235\ inches$$

Based on this calculation, it is recommended that a 20-inch parachute be used.

Calculating Descent Rate Based on Recommended Parachute Size

When we calculated the parachute size, we used a default descent rate of 3 meters per second. Now that we know we should use a 20-inch (0.508001 meters) parachute, we need to calculate the actual descent rate (v_d). The formula we use is the same as our single stage rocket.

$$v_d = \sqrt{\frac{8mg}{\pi \rho C_d D^2}}$$

$$v_d = \sqrt{\frac{8 \times 0.0831 \times 9.81}{3.1415926 \times 1.2250 \times 0.75 \times 0.508001^2}}$$

$$v_d = \sqrt{\frac{6.521688}{0.7448629}}$$

$$v_d = \sqrt{8.7555549}$$

$$v_d = 2.9589787 \; meters\ per\ second$$

Calculate Descent Time

Now that we have calculated the rate of descent with our 20-inch parachute, we can calculate the descent time (td). To determine the number of seconds it will take to reach the ground, divide the altitude at apogee by the parachute descent rate.

$$t_d = \frac{Total\ Altitude}{v_d}$$

$$t_d = \frac{372.0188355}{2.9589787}$$

$$t_d = 125.7254185 \; seconds$$

$$t_d = 2 \; minutes, 6 \; seconds$$

Sample Flight Profile Report

Below is a sample Flight Profile Report on the Farside-X rocket based on the calculations we have just completed. Such a report provides a comprehensive picture of how the model is predicted to perform during an actual flight. Remember to expect a difference of about 10% between predicted performance and actual performance.

Predicted Flight Profile Report

Launch Vehicle Data

- Name: Farside-X
- Length = 63.5 cm (0.635 m)
- Diameter = 41.6 mm (0.0416 m)
- Mass (Empty) = 71 g (0.071 kg)

Motor Information

- First Stage = B6-0
- Average Motor Thrust = 6 Newtons
- Total Thrust = 5 Newton-seconds
- Motor Mass = 15.7 g (0.0157 kg)
- Propellant Mass = 6.5g (0.0065 kg)
-
- Second Stage = C6-0
- Average Motor Thrust = 6 Newtons
- Total Thrust = 10 Newton-seconds
- Motor Mass = 21.4 g (0.0214 kg)
- Propellant Mass = 12.2 g (0.0122 kg)
-
- Third Stage = C6-7
- Average Motor Thrust = 6 Newtons
- Total Thrust = 10 Newton-seconds
- Motor Mass = 24.3 g (0.0243 kg)
- Propellant Mass = 12.2 g (0.0122 kg)

Constants Utilized in Calculating This Predicted Flight Profile

- Air density (ρ = 1.2250 kg/m3),
- Launch Vehicle Coefficient of Drag (Cd = 0.75)
- Parachute Coefficient of Drag (Cd = 0.75)

Launch Vehicle Flight Performance

	Altitude		Velocity (m/s)		Time (secs)
Stage	*meters*	*feet*	*achieved*	*terminal velocity*	*per stage*
1	12.4722118	40.9188324	29.3460319	87.0709637	0.83333333
2	91.7380939	300.9743385	73.3741394	88.7290256	1.6666667
3	138.5331223	454.4944676	88.3331969	90.6041445	1.6666667
Coasting	129.2754075	424.1267569	N/A	N/A	4.3557515
Total	372.0188355	1220.519396			

Additional Times
- Launch to Apogee: 8.5224182 secs
- Apogee to Touchdown: 125.7254185 secs
- Total Flight Time: 134.2478367 secs (approximately 2 minutes, 14 seconds)

Recommendations
- Delay time on Third Stage Sustainer Motor: 5 seconds
- Parachute Size: 0.508001 meters (20 inches)

-------------- *End of Report* --------------

By completing a report such as this, you have an idea of how well your rocket will perform. It provides a guide that you can use to adjust inputs such as motor delay times and parachute size. The downside of these calculations is that they are very labor intensive when performed using paper and pencil. If you make a simple change (such as changing the first stage motor) then all of the calculations have to be performed again.

Fortunately today there are a several software programs that will perform these calculations for you. Different programs may use different formulas or provide additional options. If you are up for a challenge, you can tackle developing your own software that will do the calculations for you. You don't need anything special. You can begin by simply using a spreadsheet.

Comparing Simulations

The chart below shows the simulation results from OpenRocket. It was run using the same engine combination as in this Supplemental Tech Report

Tech Report 9s Configuration

Altitude	600 m
Flight Time	157 s
Time to Apogee	10.8 s
Optimum Delay	6.34 s
Velocity off Pad	11.5 m/s
Max Velocity	133 m/s
Velocity at Deployment	13.2 m/s
Landing Velocity	4.05 m/s

Motor	Avg Thrust	Burn Time	Max Thrust	Total Impulse	Thrust to Wt	Motor Wt	Size
C6	4.81 N	1.83 s	14.1 N	8.82 Ns	7.34:1	10.8 g	18/70 mm
C6	4.81 N	1.83 s	14.1 N	8.82 Ns	4.97:1	10.8 g	18/70 mm
B6	5.23 N	0.826 s	12.1 N	4.33 Ns	4.24:1	5.6 g	18/70 mm
Total:				22 Ns	4.24:1	27.2 g	

As you can see, there are some differences. The chart below highlights just a couple of these differences

	Tech Report 9s	OpenRocket
Length (centimeters)	63.5	62.2
Diameter (centimeters)	4.16	4.16
Mass, empty (grams)	71	61.2
Apogee (meters)	372	600

As this chart is beginning to show there are some discrepancies between the two versions and theyt start with the length and empty weight of the rocket. Our dimensions and weight came from the catalog, while OpenRocket uses the dimensions of the 'assembled' rocket. and each individual component weight. Another item that is different is the drag factor. OpenRocket has all the exterior parts as being 'polished' which has a much lower drag factor than the 'average' C_d of 0.75 used in this tech report.

Another difference between the two is the data used for the motors. In our simulation we used the default performance figures based on the catalog, whereas OpenRocket uses the actual thrust curve.

So which one is correct and which one is wrong? Well, they both are! There are a number of factors that can affect the overall performance of a rocket and we just touched on only a few of them. To get more accurate simulations, you need accurate inputs. What is the real weight of the rocket? How much drag is actually produced. How do the engines actually perform.

Finally, build and fly the model and compare the actual results to the simulation. What changes can be made to the simulation inputs that result in the output being more in line with the actual performance of the rocket.

If You Enjoy Rocketry, Consider Joining the NAR

If you enjoy model rocketry and projects such as the Arduino Launch Control System, then consider joining the National Association of Rocketry (NAR). The NAR is all about having fun and learning more with and about model rockets. It is the oldest and largest sport rocketry organization in the world. Since 1957, over 80,000 serious sport rocket modelers have joined the NAR to take advantage of the fun and excitement of organized rocketry.

The NAR is your gateway to rocket launches, clubs, contests, and more. Members receive the bi-monthly magazine "Sport Rocketry" and the digital NAR Member Guidebook—a 290 page how-to book on all aspects of rocketry. Members are granted access to the “Member Resources” website which includes NAR technical reports, high-power certification, and more. Finally each member of the NAR is cover by $5 million rocket flight liability insurance.

For more information, visit their web site at https://www.nar.org/